Numerical Study of cusp and air entrainment regime

Localised Direct Numerical Simulations of interfacial singularity profiles

ENERGY AND INDUSTRY OF THE FUTURE

GI EIF CONFERENCE 2026

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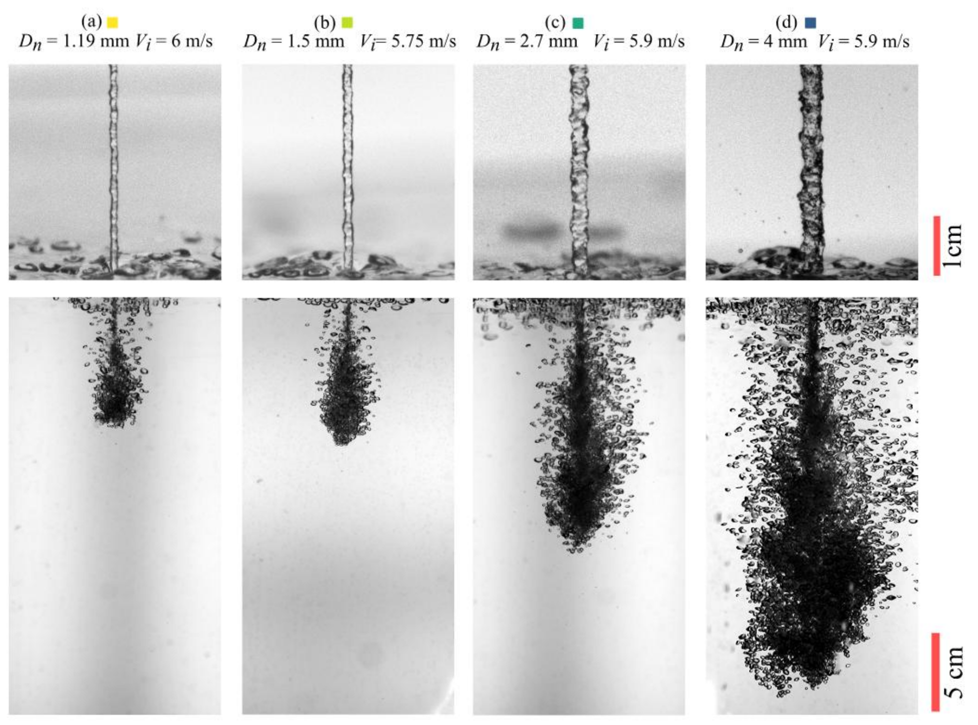
Air entrainment by plunging jet: context



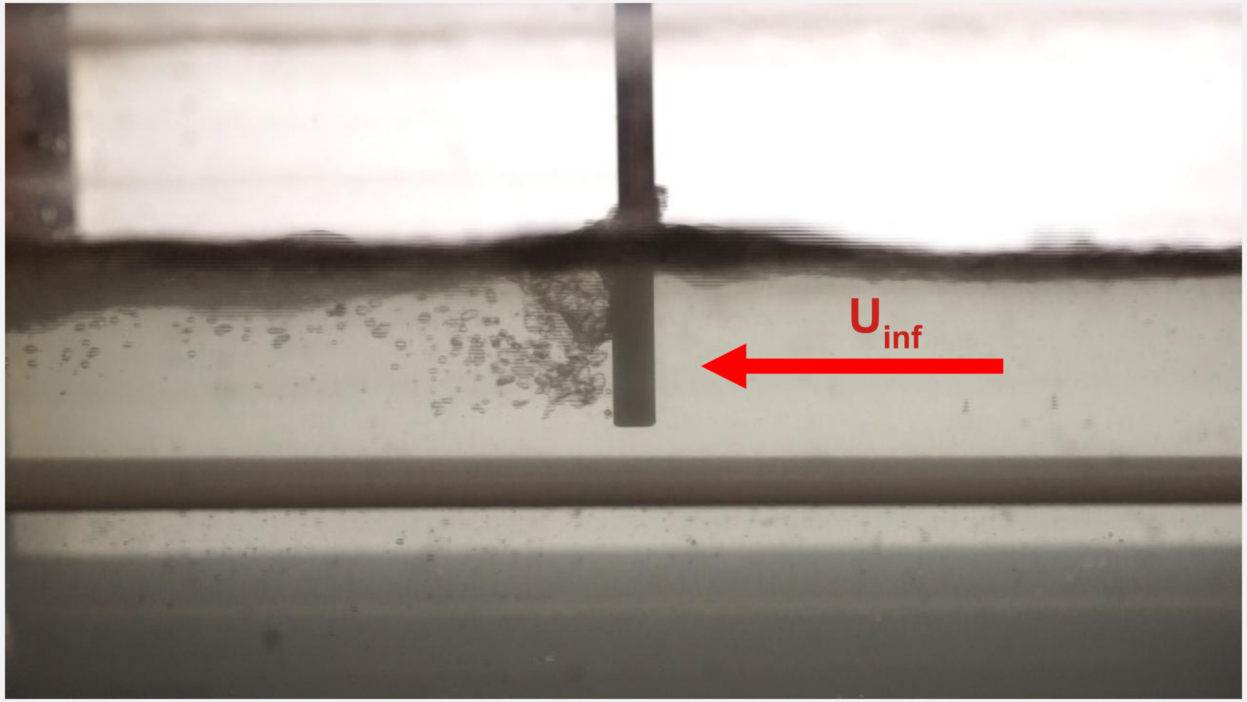
J.J.S. Jerome   H. Scolan   J.P. Matas

Inertia-dominated regime   Re ~ 10^3 à 10^4



Bubble cloud formation induced by a liquid jet impact on a free surface

Narendra Dev, PhD student (2022-2025)
ANR JetPlume

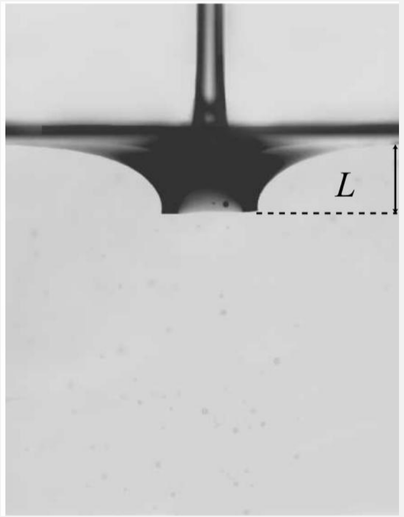


Air entrainment in the wake of a cylinder piercing a free surface

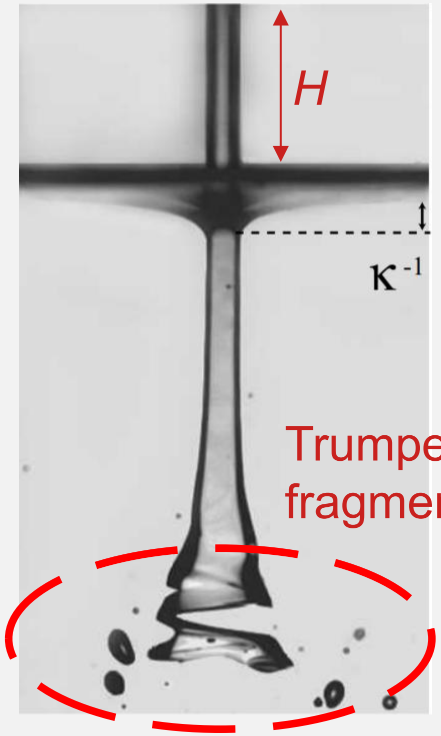
Cédric Bekale, PhD student (2025-2028)
DGA ASTRID FOAMWAKE

Air entrainment by plunging jet: context

Air Entrainment in Non-Inertial Flow Regimes Re ~ 1 à 10



Inverted meniscus: no air entrainment



trumpet: air film entrainment

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u}\right) = -\nabla P + \nabla\cdot\left[\mu\left(\nabla\mathbf{u} + {}^{t}\nabla\mathbf{u}\right)\right] - \sigma\kappa\delta\mathbf{n} + \rho\mathbf{g}$$

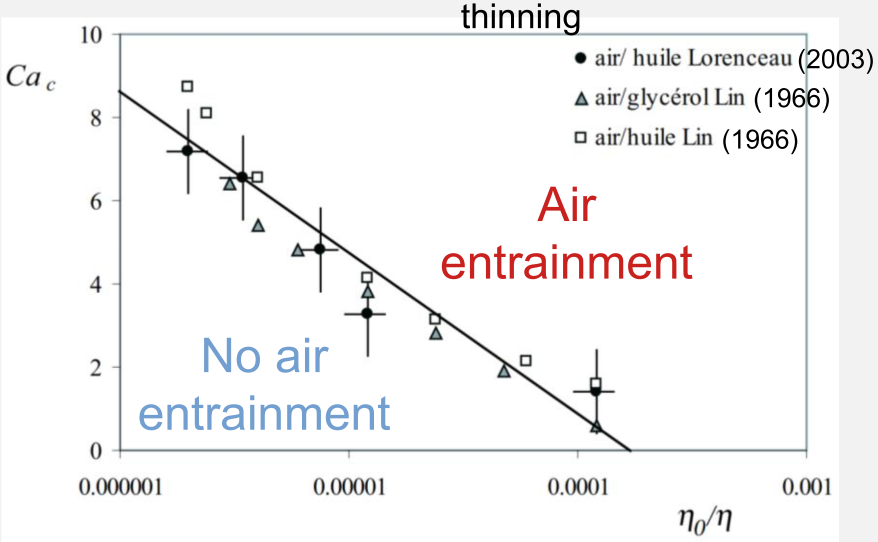
Inertial terms — Viscous terms — Capillary terms — Gravity

$$Ca = \frac{\text{viscous terms}}{\text{capillary terms}} = \frac{\mu_l U}{\sigma}$$

Capillary number

$$Fr = \frac{U}{\sqrt{gH}} > 1$$

Froude number (>1) : to prevent jet acceleration and thinning



E. Laurenceau, PhD thesis, 2003

Context

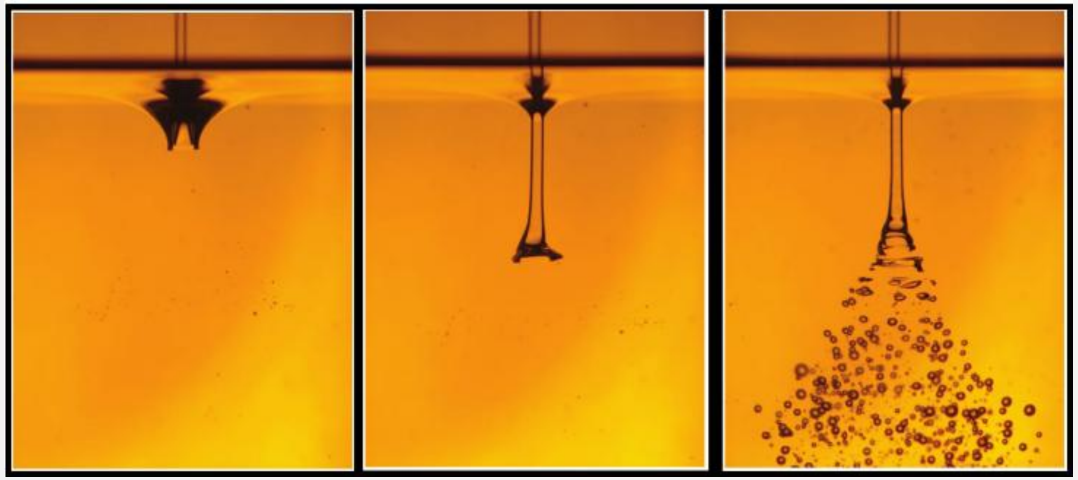


Fig 1: A plunging viscous jet forming a cusp at the interface (left).
As the Ca increases, the film of air penetrates into the bath (centre)
This film of air breaks into bubbles in the bath
[Reyssat et al. (2008)]

Viscous jet plunging into same liquid curves the interface into a cusp.

↓

Beyond critical Ca: cusp ruptures, air film penetrates forming a trumpet.

↓

Trumpet destabilises into bubbles descending into the bath.

Three regimes, one control parameter:

$$\textit{Capillary Number (Ca)} = \frac{(viscosity(\mu) \times fluid\ velocity\ (U))}{surface\ tension\ (\sigma)} = \frac{Viscous\ drag}{surface\ tension}$$

Literature: Eggers, Lorenceau and Reyssat

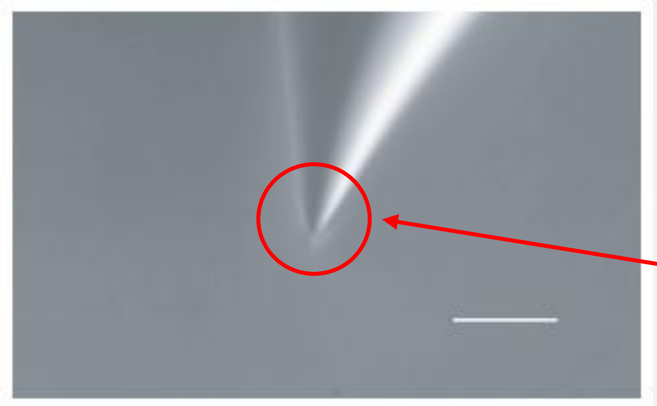


Fig 2: Photograph of the air meniscus forming a cusp into glycerol. [Lorenceau (2003)]

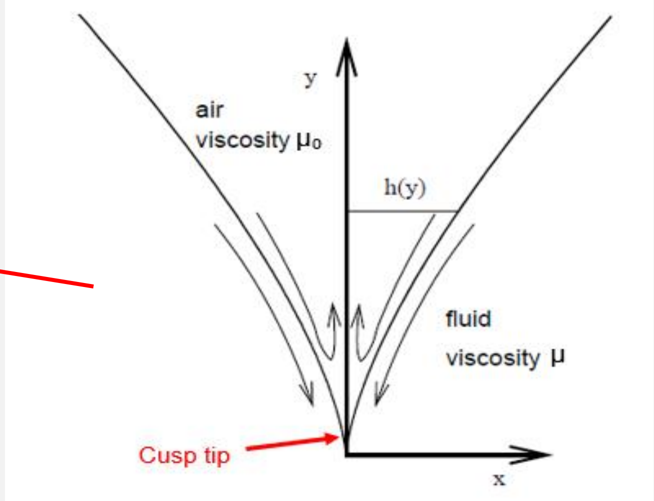


Fig 3: Local profile of cusp, cut perpendicular to the sheet of air. [Eggers (2000)]

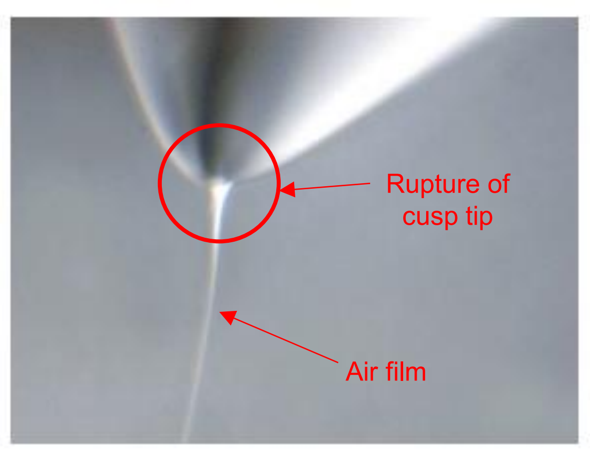


Fig 4: Beyond a threshold Ca, tip breaks and a thin layer of air enters the bath of glycerol. [Lorenceau (2003)].

Fig 5: Jet of air break-up, generating bubbles in the bath. [Reyssat (2007)].

➔ Fig 2: Stationary and stable cusp (low Ca):
Tip radius $r \propto \exp(-Ca)$ [Jeong & Moffatt (1992)].

➔ Fig 4: Rupture ($Ca > Ca_c$). Critical threshold for air entrainment:
$Ca_c \propto (2/3\pi) \ln(\mu/\mu_0)$ [Eggers (2001)].

➔ Fig 5: The air sleeve/trumpet destabilises into bubbles, consistent with a Plateau-Rayleigh mechanism [Reyssat (2007)].

Basilisk Solver

(Popinet, S., http://basilisk.fr)

What is Basilisk?

➔ Open-source C framework for incompressible multiphase DNS on adaptive quadtree grids.

➔ Built for free-surface and interfacial flows; solves Navier-Stokes with VOF and AMR natively

[Popinet (2009), (2015)]

Volume of Fluid

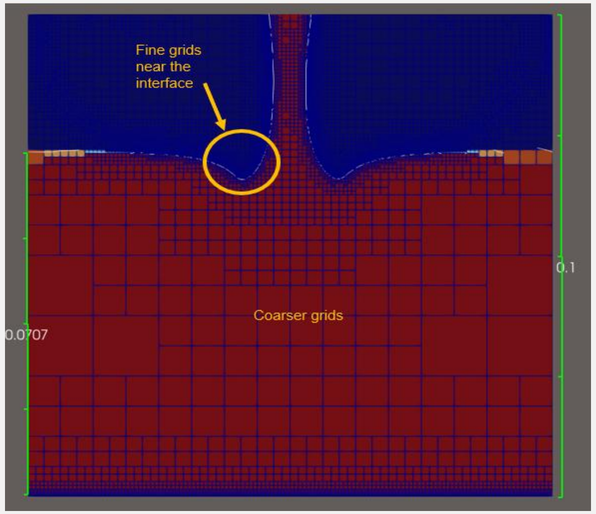
➔ Tracks interface via scalar f ∈ [0,1]; f=1 liquid, f=0 gas

➔ VOF + conserving.h is essential here given density ratios up to 1000:1, where standard advection produces spurious velocities.

Adaptive mesh refinement

➔ Refines cells only where needed (interface, jet) each timestep, coarsens bulk regions.

➔ “adapt_wavelet” estimates error on f and u, refines if above 1% of U_{max}



Methodology:

Basilisk DNS: VOF (sharp interface) + AMR (resolves thin air film)

Setup Strategy

Achieving a physical cusp was not immediate. We started from an atomisation code and executed a multi step parameter search to trigger the target regime.

The goal is to find the threshold Capillary number (Ca) at which cusp tip ruptures and the air entrains into the bath. To navigate this search, we focus on three dimensionless numbers at the nozzle.

1. $Re = \frac{\rho U R}{\mu} = \frac{inertia}{viscous\ forces}$
2. $We = \frac{\rho U^2 R}{\sigma} = \frac{inertia}{surface\ tension}$
3. $Ca = \frac{\mu \times U}{\sigma} = \frac{viscous\ drag}{surface\ tension}$

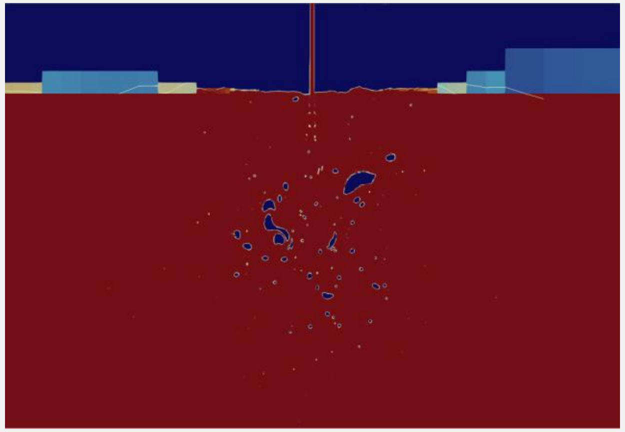


Fig 6: Initial atomisation visuals from starting codebase (Re 4000, We 161.5)

- Reduced velocity to transition away from aggressive atomisation
- Adjusted fall height to stabilise gravitational influences relative to surface tension
- Varying physical domain size and diameter of the jet
- Balanced key dimensionless numbers (Re, We, Ca) to get a stable attached flow

Exploration Phase

Approach

➔ Once the attached flow regime solidified, we tested different We (Weber number) and Re (Reynolds number) combinations to capture stable cusps, air trumpets and bubble detachments.

➔ These two bracketed the physics, all subsequent work stays between them.

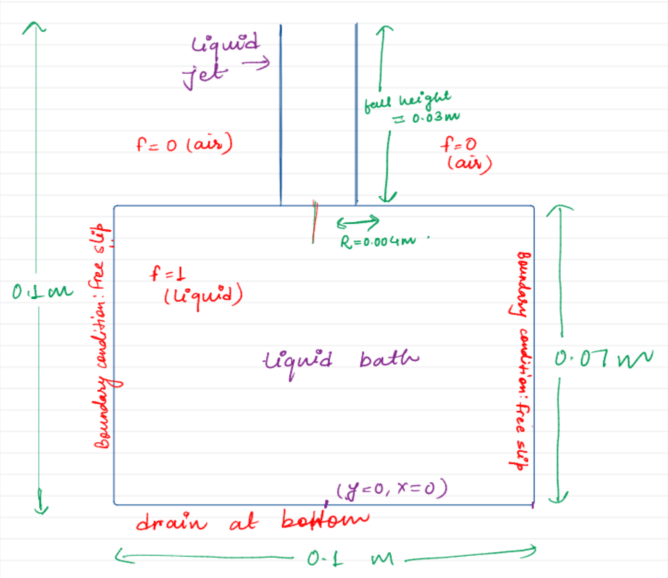


Fig 7: Representative sketch of the setup

Box size = 0.1 m x 0.1m
Diameter of the jet = 0.008 m
Fall height = 0.03 m

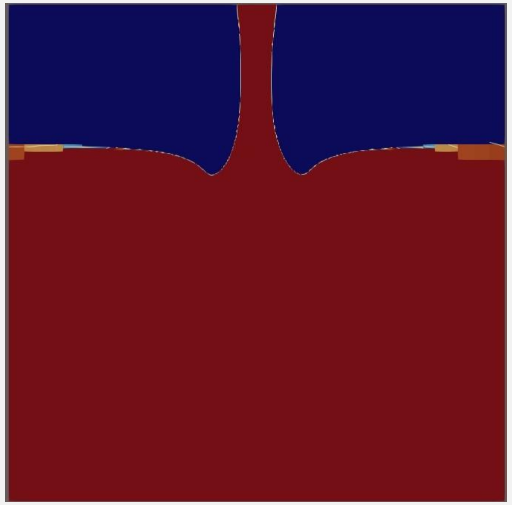


Fig 8: Cusp (Re 0.6, We 2.6, Ca 4.3)

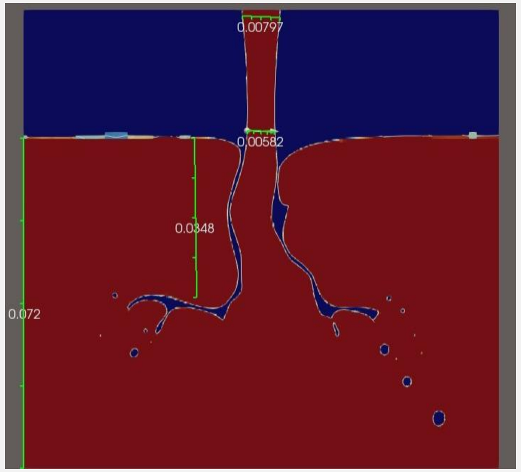


Fig 9: Bubbles (Re 1.07, We 13, Ca 12.2)

Fig 8 and 9 showing two extremes of the domain (cusp and the breakup of air film into bubbles) as the Ca increases

Finding the Entrainment Threshold

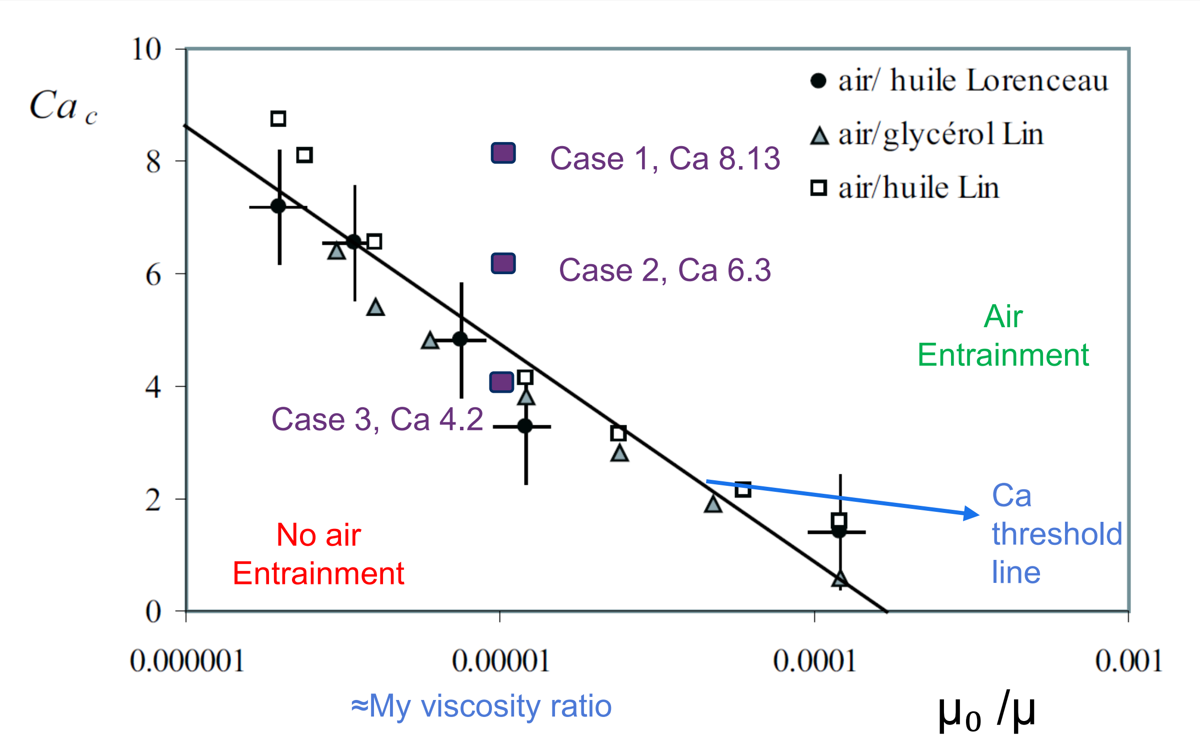


Fig 10: My results (points) compared to Lorenceau's jet threshold law (line). Viscosity ratio μ_0 / μ air over liquid), log scale, running from 10^{-6} to 10^{-3}

Results visualised in Paraview

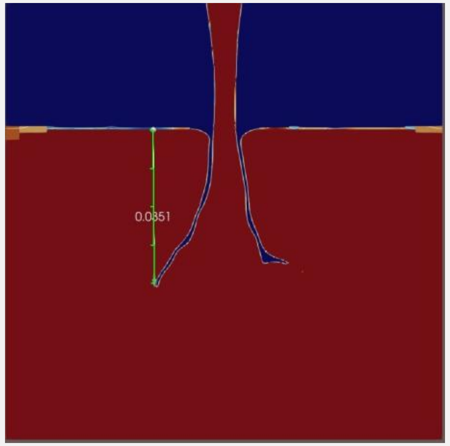


Fig 11: Case 1 (Re 0.71, We 5.8, Ca 8.13)

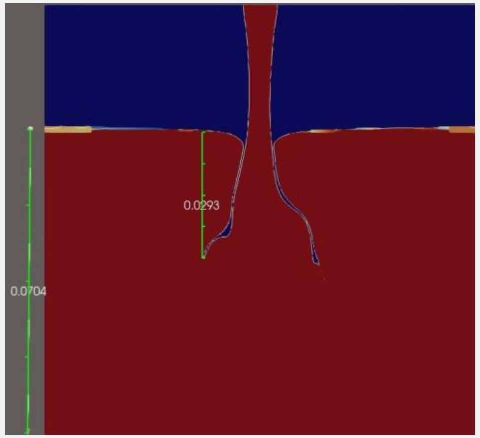


Fig 12: Case 2 (Re 0.71, We 4.5, Ca 6.3)

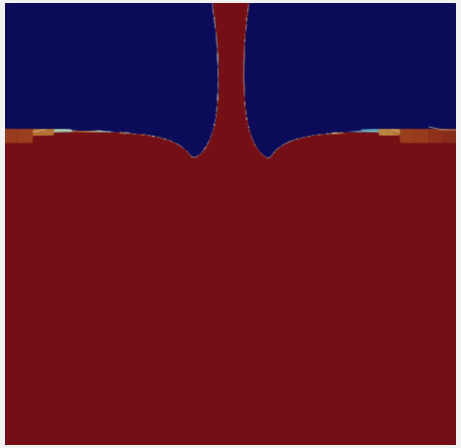


Fig 13: Case 3 (Re 0.71, We ~ 3, Ca 4.2)

Hypothesis and further work

Analysis

➔ Lorenceau's modified threshold at nozzle predicts $Ca_c \approx 4.5$ for our viscosity ratio; lies within our bracketed range (4.2–6.3).

➔ Impact-condition Re, We, Ca differ from injection values due to gravitational fall; quantified directly in our simulations (Slide 7).

Thickness of the air film

➔ The thickness of the entrained air film (e) is governed by the jet's radius (R) and the Capillary number [Reyssat (2007)].

$$e \cong 1.3\, R\, Ca_o^{2/3}$$

➔ Reyssat's experimental result, a small-jet adaptation of Landau-Levich result where the jet radius R replaces the capillary length.

Work in Progress

➔ Extend simulations to 2D axisymmetric geometry, closer to the physical (round jet).

➔ Systematic parameter sweep on clean fluid line → robust Ca_c with clear Re, We dependence.

➔ Grid convergence study targeting air film thickness and trumpet length.

➔ Assess validity of jet adaptation of Landau-Levich result for film thickness at high Ca.

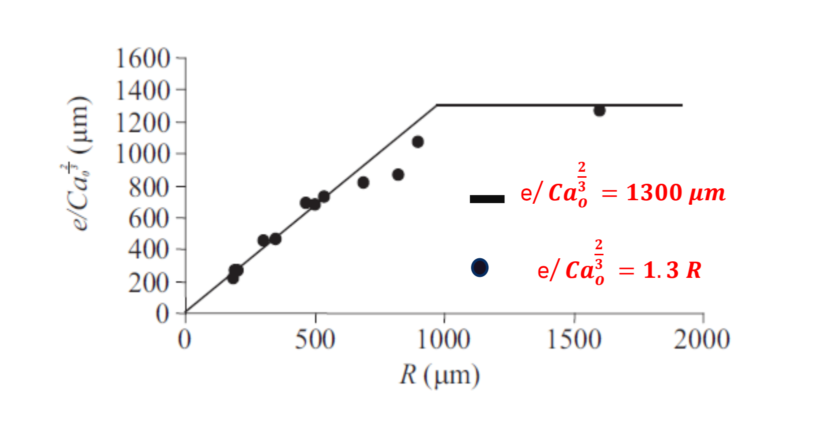


Fig 14: Film thickness as a function of jet radius [Reyssat (2007)]

THANK YOU